



## Beautiful Geometric Origami Models (2D and 3D) – Mosaics and Polyhedra

Wunderschöne geometrische Origami Modelle (2D und 3D) – Mosaike und Polyeder  
Formy geometryczne płaskie i przestrzenne w technice origami modułowego

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Mathematics Teacher Association

Galician Group of Origami

Krakow, Poland

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## Beautiful Geometric Origami Models (2D and 3D) – Mosaics and Polyhedra

We will analyze geometry of A4 sheet of paper at first.

Then we would show some examples mosaics prepared in origami.

We will discuss polyhedron properties with origami.

Als erstes werden wir die Geometrie des A4-Blattes analysieren.

Dann werden wir einige vorbereitete Mosaikbeispiele im Origami zeigen.

Es werden polyedrische Eigenschaften mit Origami diskutiert.

Im gesamten Workshop wird Material gezeigt und mit den Teilnehmern selbst gefaltet.



### About us

We are both mathematicians.

When we met origami and its geometric models our mathematical thinking changed. We understand geometry and geometrical construction much better.

When we teach origami models we underline the relations between origami and geometry. In our opinion origami in mathematics' education process could be an interesting experience, both for the teacher and students.



### Terminology notes

1. We make geometric constructions and we use mathematical terms when we fold.
2. We will use mathematical language to describe folding process as I normally do in the classroom.
3. When we say "square" we know that it is abbreviation for "a sheet of paper (or a part of it) which has square shape".



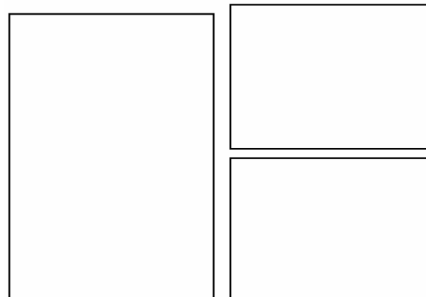
## Planar geometry concepts in origami

1. Origami is a part of geometry - we make geometric constructions and we use mathematical terms when we fold.
2. Origami may be described by geometry - we use mathematical language to describe folding process.



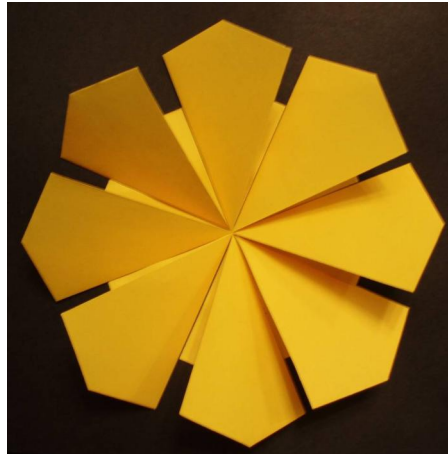
## A4 paper

Divide an A4 sheet of paper into halves by a line parallel to the shorter side.





## Deltoids



## Problem - On the Tiles

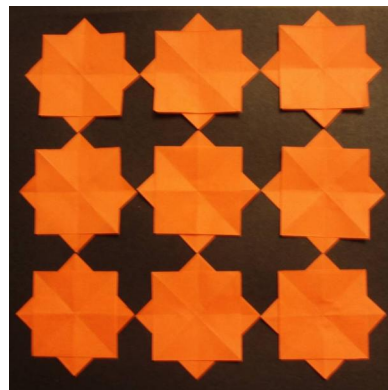
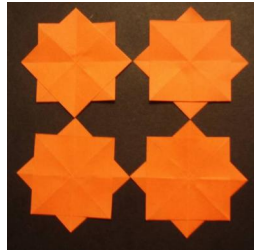


During An archeological „dig” At the monastery of San Fruttuoso near Genoa, the found a tiled floor made from an equal number of two different kinds of tiles. One kind was a regular eight-pointed star that is made by taking two squares of side 10 cm and putting one on top of the other, making sure that the centers of the squares are also one on top of the other. The other kind of tile has the same length of perimeter as the first type. This second kind fits with the first to form a tiled floor without any gaps.

Math on the Move, December 2002

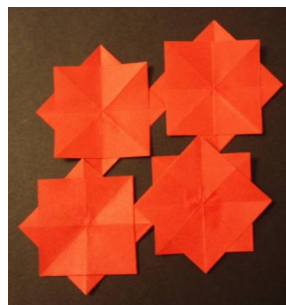


## Problem - On the tiles - Solving



## Problem - On the tiles

Another two tillings, but these do not held problem conditions.



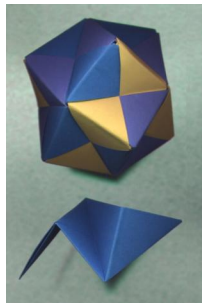


## Modular origami



### Modular origami

Modular origami models made of many, usually identical modules. Modules are joined together without a glue or thread to form more complex geometrical structures, both flat or spatial.

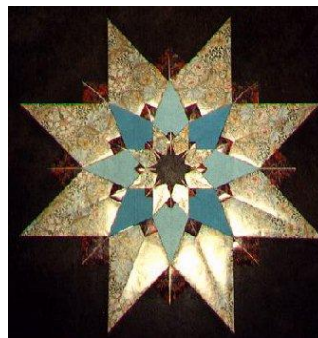




## Mosaics

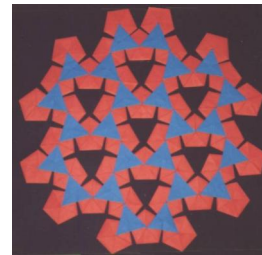
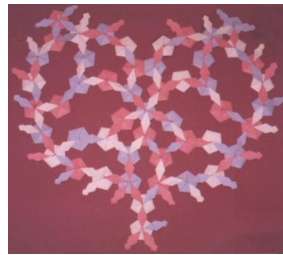
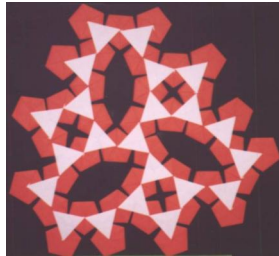


### Example - Radial Mosaic (creator Tomoko Fuse)





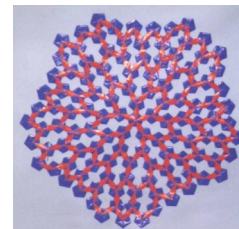
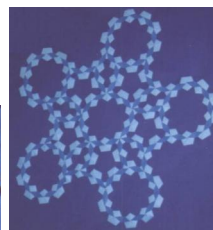
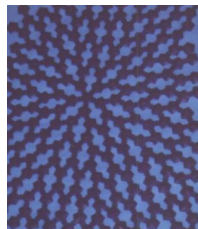
### Example - Pentagonal mosaics



Models folded by students of high school in Bochnia, Poland  
(I Liceum Ogólnokształcące w Bochni).



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Models folded by students of high school in Bochnia, Poland  
(I Liceum Ogólnokształcące w Bochni).





## Polyhedral structures

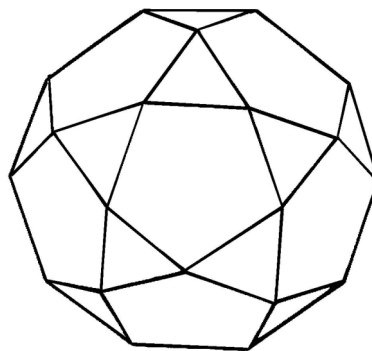


### Basic concepts and terms

Truncated  
dodecahedron

Faces:  
12 pentagons  
20 triangles

Edges: 60  
Vertices: 30





## Origami models of polyhedra

Origami polyhedra models divide into three groups based on the technique used to connect modules:

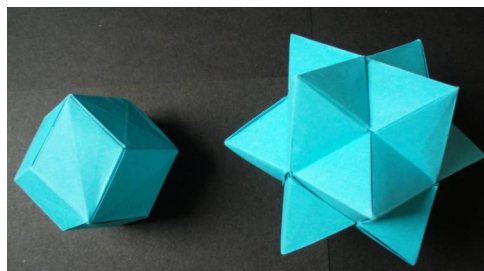
Face models

Edge models

Vertex models



## Face models - Rhombic Dodecahedron and Stellated Dodecahedron



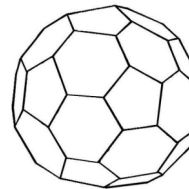
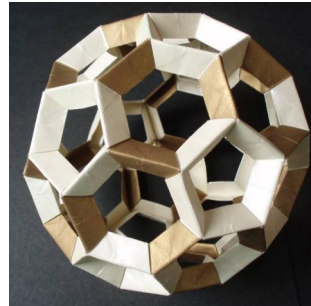
Pyramids  
– packing  
the space



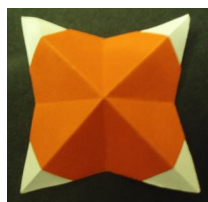


## Edge model - Truncated icosahedron

Model made of 90 edge modules.

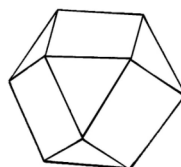


## Vertex module and cubooctahedron



Cubooctahedron

Model made of  
12 vertex  
modules.

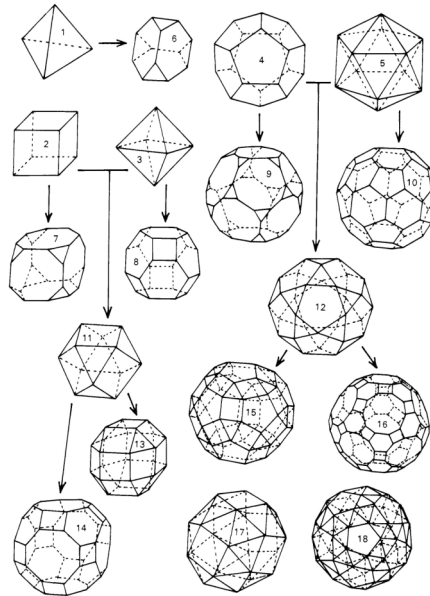
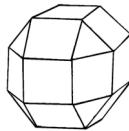




## Exercise - Twirls

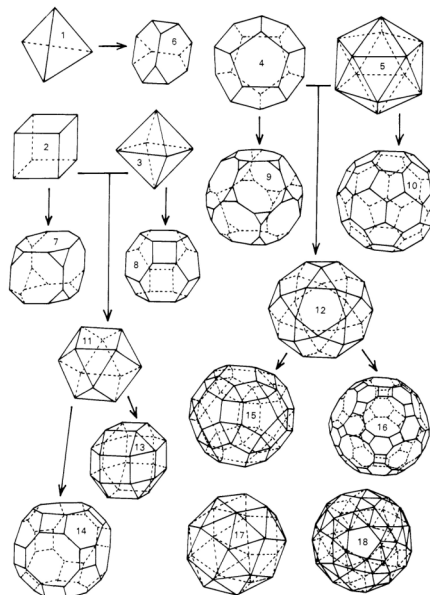
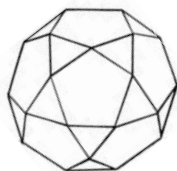


Can you see a polyhedron?



## Twirls

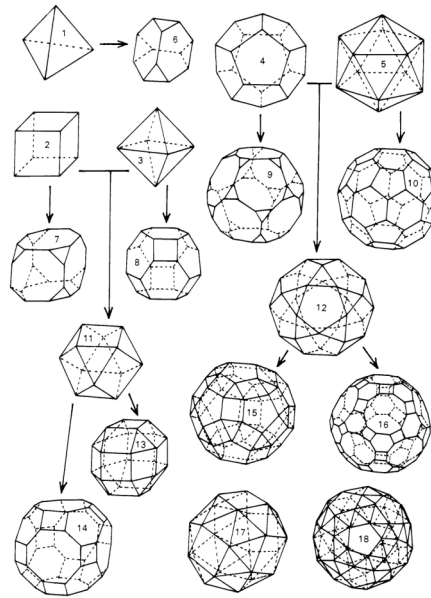
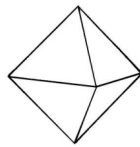
Can you see a polyhedron?





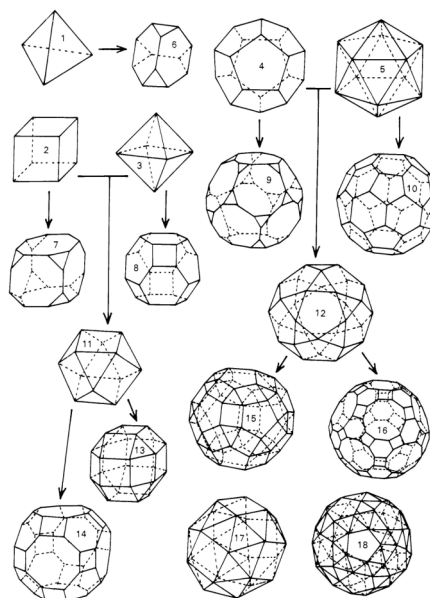
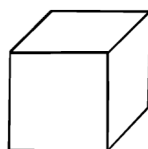
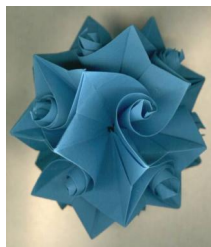
## Twirls

Can you see a polyhedron?



## Twirls

Can you see a polyhedron?

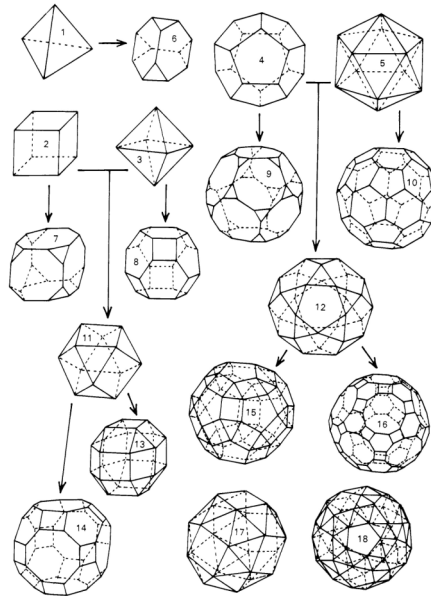
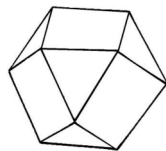




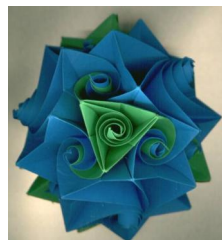
## Twirls



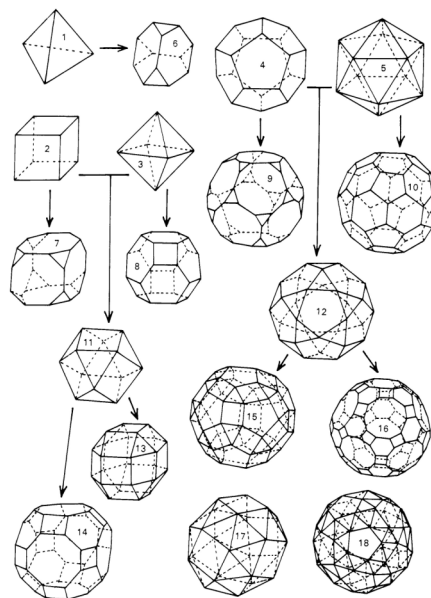
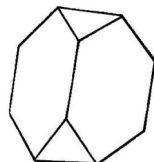
Can you see a polyhedron?



## Twirls



Can you see a polyhedron?



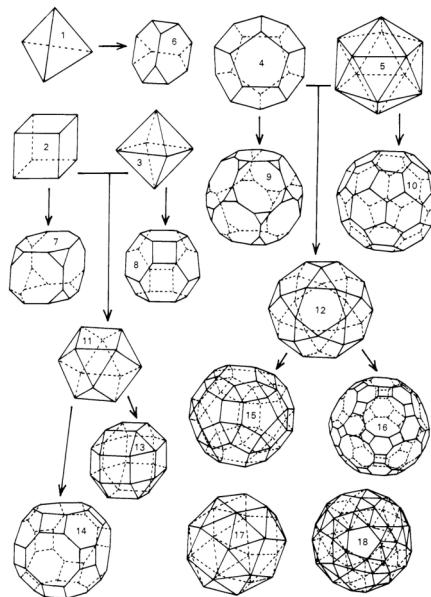




## Twirls

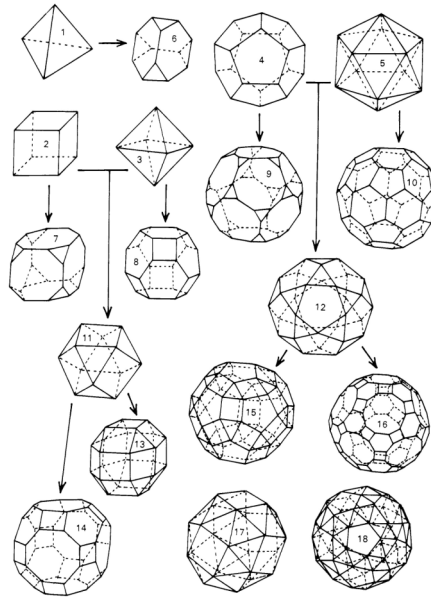
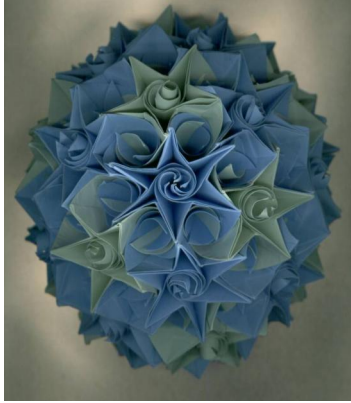


## Twirls

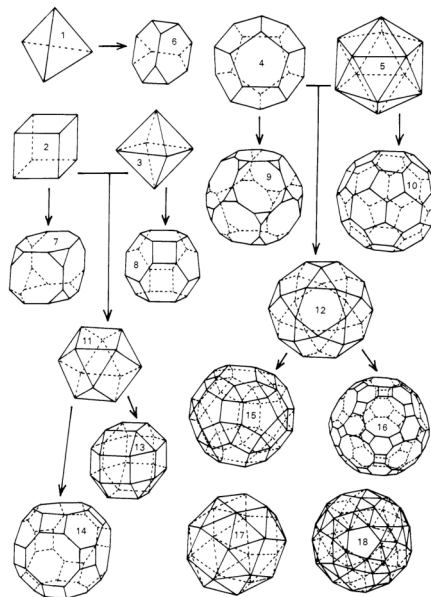
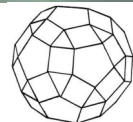
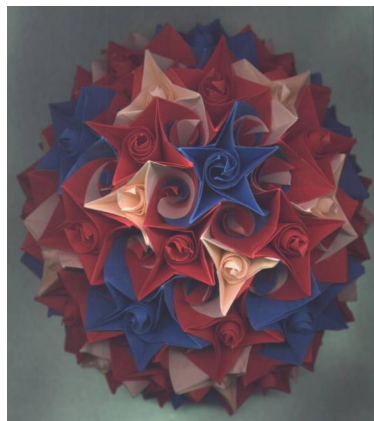




## Twirls



## Twirls







### About colors

Origami models are colorful.

We can use paper in several colors to make different colors modules for faces, edges or vertices.

So we have another group of mathematical problems:

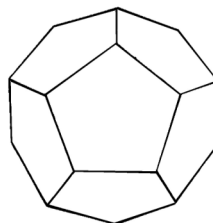
- What is minimal number of colors necessary for a model?
- How many different color patterns can be used for a model?



### About colors

Dodecahedron – model made of 12 flowers which are pentagons.

How many colors do you need?

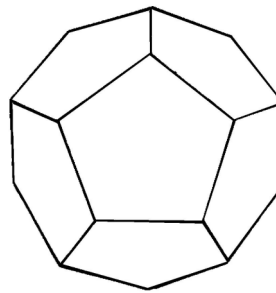




### About colors

Dodecahedron – model made of 30 modules which are edges.

How many colors do you need?



### Conclusions



## Conclusions

We can use origami model of polygon to explore its properties and to find relations to other polygons (for instance making additional folds).

An origami model of polyhedron can be used to study polyhedra properties.

We can investigate geometrical structure (relations between faces, vertices and edges, sections) and fix the geometrical concepts.

We can study relations between different polyhedra.



## Conclusions

With origami we can compose spatial (for instance pyramid) or flat (for instance mosaic) structure.

Such activity develop spatial imagination and recognition - elements are manipulated in the space, rotated and translated, different (not only orthogonal) orientation of objects and rotation axis are introduced naturally.



**Thank You!**