



Origami as didactic tool in mathematics education

**Origami als didaktisches Werkzeug im Mathematik.
Origami jako narzędzie dydaktyczne w kształceniu
matematycznym.**

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Mathematics Teacher Association
Galician Group of Origami
Krakow, Poland

17. November 2006 – Friday – Freitag, 18:30-20:00



Origami as didactic tool in mathematics education.

- | | |
|--|---|
| – Geometric concepts and constructions in geometry of origami. | – Unterricht Geometrische Konzepte und Konstruktionen in der Geometrie des Origami. |
| – Communication. | – Kommunikation. |
| – Mathematical language. | – Mathematische Sprache. |
| – Visible and hidden levels of teaching process. | – Sichtbare und versteckte Schwierigkeitsgrade im Unterrichtsverfahren. |



About us

We are both mathematicians.

When we met origami and its geometric models our mathematical thinking changed. We understand geometry and geometrical construction much better.

When we teach origami models we underline the relations between origami and geometry. In our opinion origami in mathematics' education process could be an interesting experience, both for the teacher and students.



Origami as didactic tool in mathematics education

Origami creates a good opportunity to practice mathematical terms and to instantly verify students' understanding of those terms.

Origami also supports the transfer of mathematics content in the folding process, during the planning of origami work and during the problem solving process.

Origami offers a unique opportunity to structure the teaching process into two levels: visible and hidden.



Origami as didactic tool in mathematics education

We would like to show connections between origami, geometry and solving interesting problems.

We would like to illustrate the goals of the visible and hidden levels in origami and show some examples about them.



Origami and geometry

1. Origami is a part of geometry - we make geometric constructions and we use mathematical terms when we fold.
2. Origami may be described by geometry - we use mathematical language to describe folding process.



Terminology notes

1. We make geometric constructions and we use mathematical terms when we fold.
2. We will use mathematical language to describe folding process as we normally do in the classroom.
3. When we say "square" we know that it is abbreviation for "a sheet of paper (or a part of it) which has square shape".



Exercise – divide into halves

Make the exercise:

Divide a square into halves.



Exercise – divide into halves

The instruction is obvious, but ...
what does it mean?

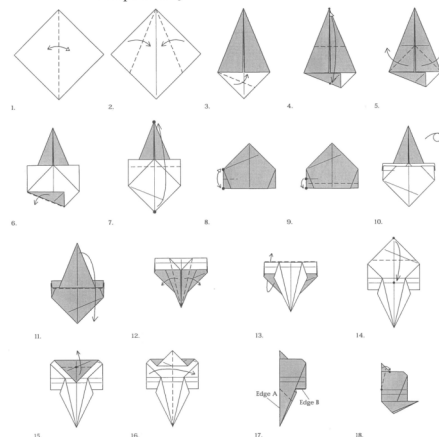
1. Fold a square along diagonal.
2. Fold a square along line parallel to side of square.
- or
3. Fold a square sheet of paper to obtain two identical parts.



Demonstration – Elephant (creator: Marc Kirschenbaum)

1. Fold a square along a diagonal.
2. Fold two angles along their bisectors.
3. You have a deltoid divided into two isosceles triangles. One of them is a right triangle. Bisect its acute angle.
4. Fold the vertex of the acute angle of a quadrilateral to the just folded line (to the intersection point of an original square diagonal and a bisector line in a triangle). Unfold.

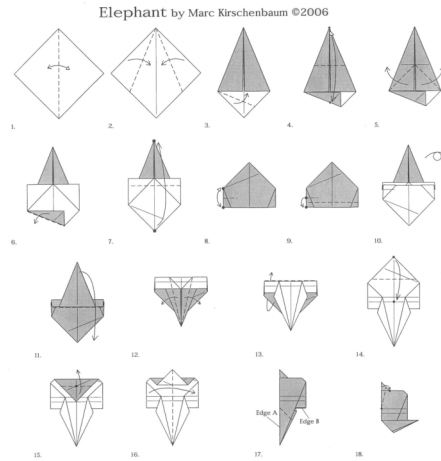
Elephant by Marc Kirschenbaum ©2006





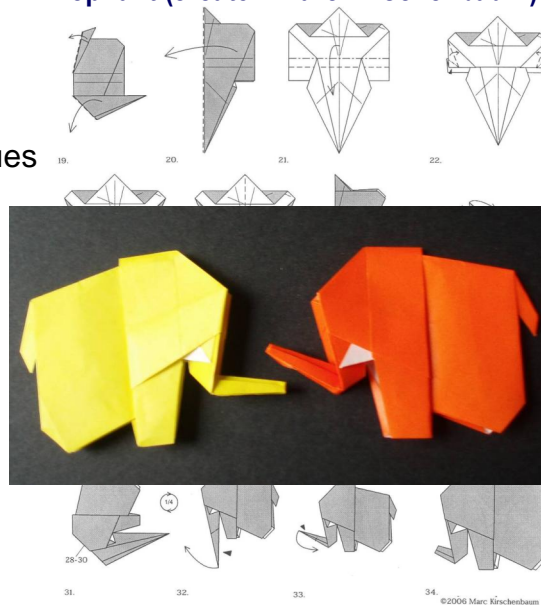
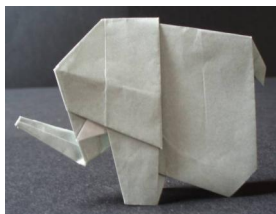
Demonstration – Elephant (creator: Marc Kirschenbaum)

5. Just folded line intersects the altitude of an isosceles triangle. Fold two lines from this point to the vertices of the triangle (ends of the base). You folded two right-angled isosceles triangles.
6. Unfold the right-angled triangle part.
7. Bisect a segment between two most distant points of a figure.
8. You have a pentagonal shape with two right angles and one axis of symmetry.



Demonstration – Elephant (creator: Marc Kirschenbaum)

Folding process continues and after sequence of geometric construction you can obtain ... a sweet elephant.





Origami offers a unique opportunity to structure the teaching process into two levels: visible and hidden.



Visible and hidden level

Two levels of the teaching process:

Visible level

- Goal for the students
- Focus on the result
- An origami model as a goal

Hidden level

- Goal for the teacher
- Focus on the process
- Development of mathematical concepts and skills and development of communication skills as a goals.



Visible and hidden level

Good use of origami for mathematics education should offer on the visible level a model which is

- visually attractive,
- relatively easy to fold
- resistant to inaccuracy

Hidden level should strictly connected with educational goals. Mathematical language should be a necessary part of the folding process and not its side-effect.



We would like to illustrate the goals of the visible and hidden levels in origami.



Example - Elephant (creator Marc Kirschenbaum)

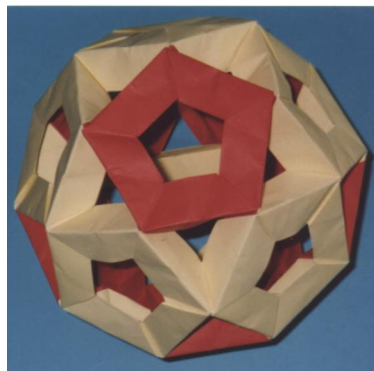


Visible level: to make an elephant.

Hidden level: to make and study of geometric constructions.



Example - Modular polyhedron made of edge modules



Visible level:

- to obtain visually attractive model.

Hidden level:

- to discover and observe symmetry of regular polyhedra.

When symmetry is not observed, the model is crooked or falls apart.



Example - Knotty problem **KNOTOLOGY**

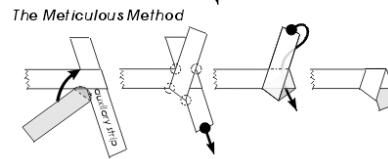
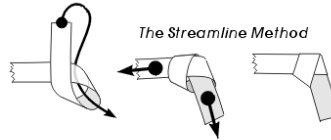
by Heinz Strobl, Knotologist

Use a strip of paper 4 cm wide to make a knot like on the diagram. The knot should look like a pentagon.

Divide the pentagon into sections, coloring in the sections which have the same number of layers of paper.

Math on the Move, March 2000

Two methods to fold a Pentagonal Knot



© Heinz Strobl

KNOTOLOGY 1+



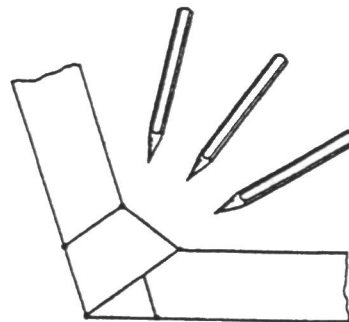
Example - Knotty problem

Visible level:

- To color a pentagon.
- To solve a puzzle.

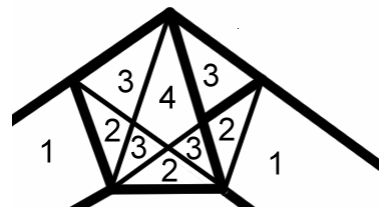
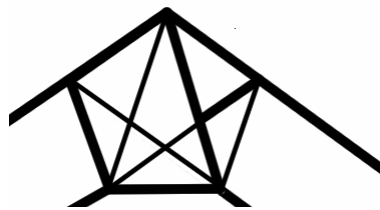
Hidden level:

- To analyze a geometric model.
- To imagine how paper goes through a knot.
- To build a mental representation of a physical model.





Example - Knotty problem - Solving

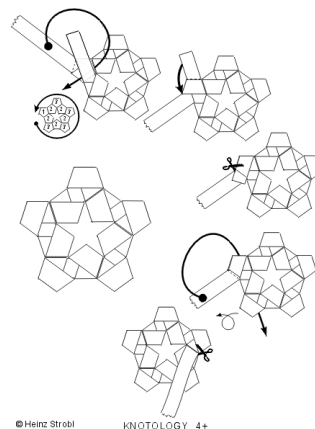


Extension to the Heiz Strobl's knotology

We can tie more than one knot on a long strip of paper.

We can even form spatial (3 dimensional) object this way.

Locking of the Star Module



© Heinz Strobl

KNOTOLOGY 4+



Extension to the Heiz Strobl's knotology



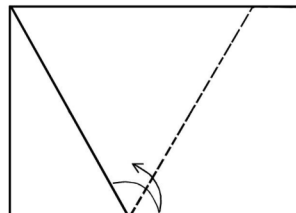
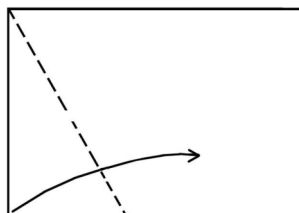
Sphere94 (Heinzl Strobl)

folded by Rosa <http://home.tiscali.nl/gerard.paula/origami/knotology.html>



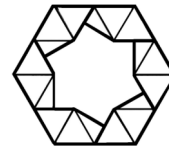
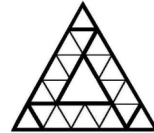
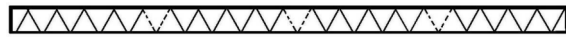
Example - Strip folding and number sequence

1. Fold a strip aslant.
2. Bisect an obtuse angle (between the fold line and the edge of the strip).
3. Bisect newly created obtuse angle (on the opposite side of the strip).



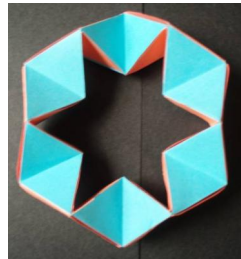
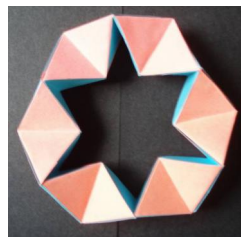
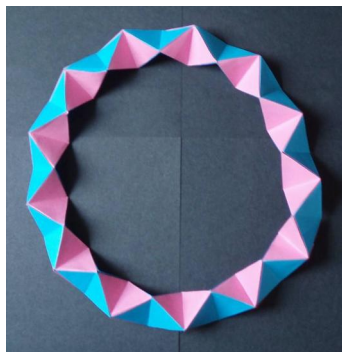


Example - Strip folding and number sequence



Example - Strip folding and number sequence

Kaleidocycle

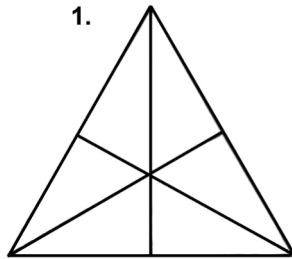




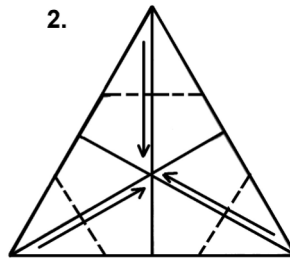
Example - Strip folding and number sequence

Star

1.



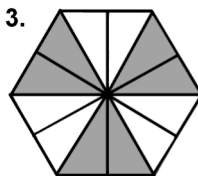
2.



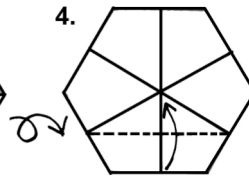
Example - Strip folding and number sequence

Star

3.



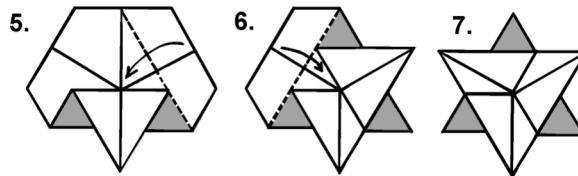
4.





Example - Strip folding and number sequence

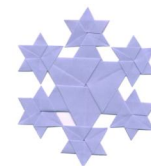
Star



Example - Strip folding and number sequence

Visible level:

- To make equilateral triangles or regular hexagons.



Hidden level:

- To analyze a sequence of angles.
- To analyze the convergence of the sequence.
- To analyze the error of the construction.



Example - Twirls

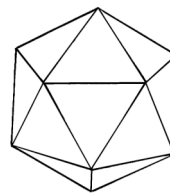
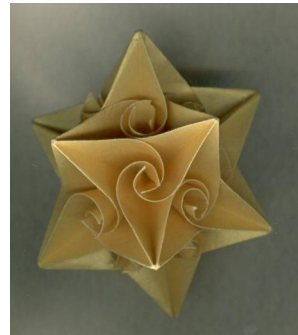
Visible level:

- To make cute color kusudama.

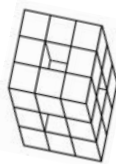
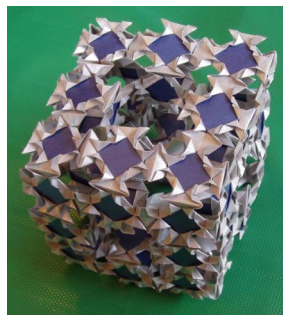
Hidden level:

- To develop and fix concepts of spatial geometry related to polyhedra.

Without understanding a geometry of polyhedra it is not possible to build a regular, spherical model.



Example - Menger Sponge, level 1



Visible level:

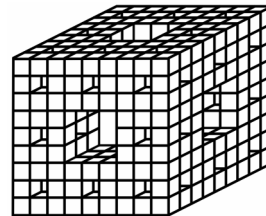
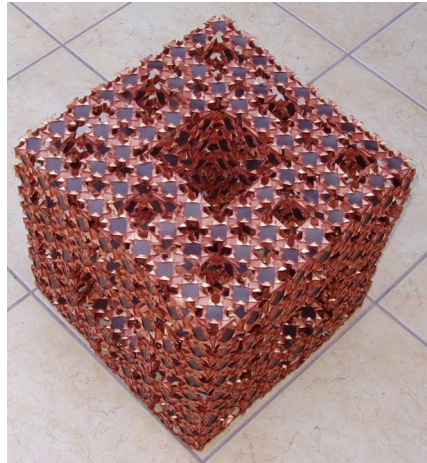
- To make an interesting geometric model as a group work - model of level 1 (or 2 or 3) Menger Sponge.

Hidden level:

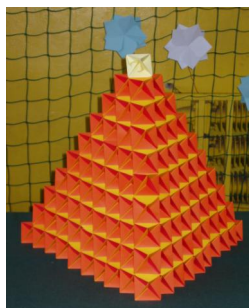
- To analyze the mathematical structure of a fractal.
- To calculate the number of pieces for a given level of a model.
- To make a model for study of recursive structure of fractal, their surface and volume.



Example - Menger Sponge, level 2



Example - A pyramid made of colored skeleton octahedra



Visible level:

- to build the pyramid as the group work outcome.

Hidden level:

- to develop spatial imagination,
- to use triangle and pyramid numbers.

When orientation of the octahedra is not observed, different colors appear. When number of pieces is not a triangle (pyramid) number, there is too many or too small pieces to construct the next level of the pyramid.



Origami in the classroom

Folding of an ordinary sheet of paper is an excellent enhancement of a lesson.

It can be employed

when we talk about polygons, their area and circumference,

when we make geometrical constructions,

when we talk about polyhedra

or

when we need a heart for the friend.

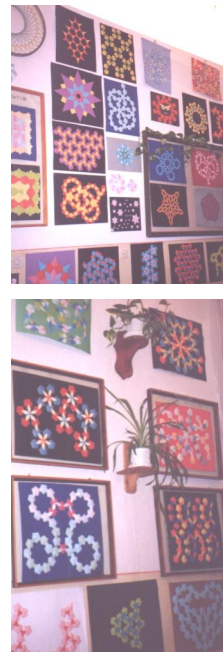


Origami in the classroom

You needn't to use origami on every geometry lesson.

Instead origami could be used:

- As an introduction to the block of lessons used to energize the spatial imagination.
- To prepare models used on the lesson.
- To decorate a classroom.
- As interesting subject for supplementary activities.





Origami in the classroom

Origami can be used in different configurations:

when only teacher folds,
when only students fold,
when they folds together under
directions of teacher or student.



And when lesson ends ...

- Student may bring a model at home.
- Student may glue a model into his/her workbook (in the case of flat models).
- A model may become a gift for somebody.
- An origami installation may be created as a group work.



Introducing origami into mathematical education enhances the mathematics language development.



Mathematical words and terms can be early introduced into vocabulary.

Introducing names of geometrical objects and properties and mathematical terms is accompanied by multi-sensored activities, which enhances their memorization.



Mathematical terms are used in the real world situation in the new context, which enhances better understanding and supports migration of such terms into student vocabulary.

The origami model could be a source of new problems, that are easy to see and understand, but must be described by students in the mathematical language in order for the problems to be solved.



Origami gives opportunity to introduce mathematical concepts during the folding process (without a formal definition, but through correct use of terms).

Understanding the meaning of geometric terms and relationships involves more than memorizing definitions. To fully understand the meaning, students need to demonstrate, manipulate, and communicate applications in variety of settings. [Cipoletti 2004]

Teacher should use mathematical language and should support students' use of mathematical language through the requirement of precise (or more precise) statements.



Verbal directions

Verbal directions can be constructed in different ways:

by the name of the folding line, student determine where the line should be folded, for example:

- fold along the bisector of the right angle
- fold along the segment bisector
- fold along the diagonal

by the dynamic description of paper movement, for example:

- fold point A to point B

by the static description of the final state, for example:

- divide square into two triangles
- divide an angle into halves



When teachers are convinced to use origami
and

when they are supported by the efficient
methods for origami in the mathematics
language

it can have a positive impact on mathematics
teaching at all levels of education.



Could origami become a standard tool for teachers?

In our opinion, origami definitely could be a valuable and widely used tool for teachers, and especially maths teachers. When properly used to aid teaching of mathematics, origami gives the following benefits:

- enhances motivation of students and their interest in mathematics;
- develops spatial imagination;
- improves perception and ability to describe the relation of objects, both on the plane and in space;
- develops creativity and ability to overcome difficulties;
- enhances better understanding of mathematical ideas and better manipulation of geometrical concepts;



Origami benefits – cont.

- develops communication with mathematical language;
- develops ability to read directions in graphical form (diagrams with verbal comment or without a comment);
- develops planning of students own activity;
- enhances ability to group work and responsibility for individual work;
- enhances concentration ability and self-control;
- develops precision and freedom of expression;
- enhances self-assessment;
- develops aesthetic.



Thank You!